Machines, Buildings, and Optimal Dynamic Taxes

Appendix - NOT FOR PUBLICATION

A Differential Taxation of Capital in the U.S. Tax Code

This section first explains how the current U.S. tax system taxes returns to capital assets differentially. Then it provides a brief summary of the historical evolution of capital tax differentials.

According to the current U.S. corporate tax code, all capital income net of depreciation is taxed at the statutory rate of 35%. However, effective marginal taxes on net returns to capital might differ from the statutory rate if tax depreciation allowances differ from actual economic depreciation. To see this point, let ρ_i be the return to capital type i, δ_i be its economic depreciation rate, and $\bar{\delta}_i$ be the depreciation rate allowed by the tax code. Suppose for simplicity that at the end of a period, the firm liquidates and there is no further production. Letting τ be the statutory tax rate, the effective tax rate, call it τ_i, is given by

$$\tau_i = \frac{(\rho_i - \bar{\delta}_i)\tau}{(\rho_i - \delta_i)}.$$

As first pointed out by Samuelson (1964), if $\bar{\delta}_i = \delta_i$, then $\tau_i = \tau$. In words, when tax depreciation equals economic depreciation, then the effective tax on the return to capital i equals the statutory rate. If this is true for all types of capital, there are no tax differentials. If, instead, for capital of type i the tax depreciation allowance is higher (lower) than the actual depreciation rate, then its return is effectively taxed at a lower (higher) rate (i.e., $\tau_i < (>)\tau$). As argued by Gravelle (2003), inter alia, such discrepancies between tax depreciation allowances and economic depreciation rates across capital assets are the main cause of differential capital taxation in the United States.[22]

[22]Of course, in reality many firms continue operating for many periods and hence deduct the whole cost of

Gravelle (2003) concisely summarizes the historical evolution of tax differentials.[23] She reports that before the 1986 Tax Reform Act, the statutory corporate income tax rate was 46%. The effective tax rates on returns to most equipment assets were less than 10%, whereas they were around 35% to 40% for buildings. With the 1986 tax reform, the statutory rate was reduced to 35%, and the depreciation rules were altered to reduce the tax differentials between equipment capital and structure capital. These policy changes resulted in effective equipment capital tax rates that were about 32% and structure capital tax rates that were still about 35% to 40%. As documented by Gravelle (2011), in the current U.S. corporate tax code (which went through a minor reform in 1993), equipments are taxed at 26% on average and structures are taxed at 32%. To sum up, capital equipments have historically been favored relative to capital structures, but the difference in effective tax rates has been declining over time.

B Proofs

B.1 Proof of Proposition 2

The multipliers on period t feasibility and skilled agents' incentive constraint are denoted by $\lambda_t \beta^{t-1}$ and μ, respectively. The first-order optimality conditions with respect to $c_{s,t}$ and $c_{u,t}$ are given by, for $t \geq 1$,

$$(c_{s,t}) : (\pi_s + \mu^*) u'(c^*_{s,t}) = \lambda^*_t \pi_s \tag{B.1}$$

$$(c_{u,t}) : (\pi_u - \mu^*) u'(c^*_{u,t}) = \lambda^*_t \pi_u, \tag{B.2}$$

a capital investment over time via depreciation deductions. The differences in effective tax rates are created by tax depreciation rules that make firms depreciate their capital assets either faster or slower than their economic depreciation over time. As long as the real interest rate is positive, tax depreciation rules that allow firms to deduct depreciation faster relative to actual economic depreciation decrease the effective tax rate on capital. Inflation also affects the real value of depreciation deductions because these deductions are based on the historical acquisition costs. For a detailed description of how effective tax rates are calculated, see Gravelle (1994).

[23] For a more detailed description of how tax differentials between capital equipments and structures have changed between 1950 and 1983, see Auerbach (1983).

which imply for all $h \in H$ and $t \geq 2$,

$$\frac{\lambda_{t-1}^*}{\lambda_t^*} = \frac{u'(c_{h,t-1}^*)}{u'(c_{h,t}^*)}. \tag{B.3}$$

The first order optimality conditions with respect to the two types of capital for $t \geq 2$ are:

$$\begin{aligned}
(K_{e,t}) : \lambda_{t-1}^* &= \beta \left[\lambda_t^* \tilde{F}_2(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) + X_t^* \right], \\
(K_{s,t}) : \lambda_{t-1}^* &= \beta \lambda_t^* \tilde{F}_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*), \qquad \text{where} \\
X_t^* &= \mu^* v' \left(\frac{l_{u,t}^* w_{u,t}^*}{w_{s,t}^*} \right) l_{u,t}^* \frac{\partial \left(\frac{w_{u,t}^*}{w_{s,t}^*} \right)}{\partial K_{e,t}^*}.
\end{aligned}$$

Combining the first-order conditions with respect to capital with (B.3) implies for both $h \in H$ and $\forall t \geq 2$,

$$u'(c_{h,t-1}^*) = \beta \tilde{F}_{1,t}^* u'(c_{h,t}^*), \tag{B.4}$$

$$u'(c_{h,t-1}^*) = \beta \tilde{F}_{2,t}^* u'(c_{h,t}^*) \left(1 + \frac{X_t^*}{\lambda_t^* \tilde{F}_{2,t}^*} \right). \tag{B.5}$$

Part 1. Equation (B.4) together with the definition of intertemporal wedges proves that the structure capital wedge is zero in all periods for all $h \in H$. Since $\mu^* > 0$, $X_t^* < 0$. The definition of the equipment capital wedge and equation (B.5) imply, for all $h \in H$ and $t \geq 2$,

$$\tau_{e,t}^*(h) = -\frac{X_t^*}{\lambda_t^* \tilde{F}_{2,t}^*} > 0.$$

This finishes the proof of part 1 of Proposition 2.

Part 2. Suppose a steady state of the constrained efficient allocation exists. Letting the allocation without any time subscripts denote this steady-state allocation,

$$X^* = \mu^* v'(\frac{l_u^* w_u^*}{w_s^*}) l_u^* \frac{\partial \left(\frac{w_u^*}{w_s^*} \right)}{\partial K_e^*} < 0.$$

Thus,

$$\tau_e^* = -\frac{X^*}{\lambda^* \tilde{F}_2^*} > 0. \qquad \square$$

B.2 Proof of Proposition 3

As before, the multipliers on period t feasibility and skilled agents' incentive constraint are denoted by $\lambda_t \beta^{t-1}$ and μ, respectively. Consider the first-order optimality conditions with respect to $c_{s,t}$ and $l_{s,t}$:

$$\begin{aligned}
u'(c_{s,t}^*)A_t^* &= \lambda_t^* \pi_s \\
v'(l_{s,t}^*)\left(A_t^* - \frac{B_t^*}{v'(l_{s,t}^*)}\right) &= \lambda_t^* w_{s,t}^* \pi_s, \qquad \text{where} \\
A_t^* &= (\pi_s + \mu^*) \\
B_t^* &= \mu^* v'\left(\frac{l_{u,t}^* w_{u,t}^*}{w_{s,t}^*}\right) l_{u,t}^* \frac{\partial \frac{w_{u,t}^*}{w_{s,t}^*}}{\partial L_{s,t}^*} z_s \pi_s.
\end{aligned}$$

By the first order conditions above, $A_t^* > 0$ and $\left(A_t^* - \frac{B_t^*}{v'(l_{s,t}^*)}\right) > 0$. By Assumption 3, $B_t^* > 0$. These imply

$$\tau_{y,t}^*(s) = 1 - \frac{A_t^*}{\left(A_t^* - \frac{B_t^*}{v'(l_{s,t}^*)}\right)} < 0. \qquad \square$$

B.3 Proof of Proposition 4

Part 1. Let $\lambda_t \beta^{t-1}$ and $\mu_t(h^t)$ be multipliers on period t feasibility constraint and the incentive constraint at history h^t, respectively. Under Assumptions 1 and 2, the result follows from the first-order conditions with respect to the two capital types:

$$\begin{aligned}
(K_{s,t}) : -\lambda_{t-1}^* + \beta \lambda_t^* \tilde{F}_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) &= 0, \\
(K_{e,t}) : -\lambda_{t-1}^* + \beta \lambda_t^* \tilde{F}_2(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) & \\
+\beta \sum_{\{h^t \in H^t\}} \mu_t^*(h^t) v'\left(\frac{l_t^*(h^{t-1}, h_t^o) w_t^*(h_t^o)}{w_t^*(h_t)}\right) l_t^*(h^{t-1}, h_t^o) \frac{\partial \frac{w_t^*(h_t^o)}{w_t^*(h_t)}}{\partial K_{e,t}^*} &= 0.
\end{aligned}$$

Part 2. Also, under Assumption 1, it follows directly from Golosov, Kocherlakota, and Tsyvinski (2003) that in any period $t \geq 1$ and following any history h^t, the constrained efficient allocation satisfies the following inverse Euler equation:

$$\frac{1}{u'(c_t^*(h^t))} = \frac{1}{\beta \tilde{F}_{1,t+1}^*} E_t \left\{ \frac{1}{u'(c_{t+1}^*(h^{t+1}))} | h^t \right\}. \tag{B.6}$$

Part 2(a) of the proposition follows from the definition of intertemporal wedges in equation (9), equation (B.6) and the conditional version of Jensen's inequality. Part 2(b) of the proposition follows from Part 1 and equation (9), which defines intertemporal wedges for both types of capital. □

There is a modified inverse Euler equation for equipment capital. We derive it here for the sake of completeness. It follows from the first part of Proposition 4 and equation (B.6):

$$\frac{1}{u'(c_t^*(h^t))} = \frac{1}{\beta \left(\tilde{F}_{2,t+1}^* + X_{t+1}^* / \lambda_{t+1}^* \right)} E_t \left\{ \frac{1}{u'(c_{t+1}^*(h^{t+1}))} | h^t \right\}. \tag{B.7}$$

B.4 Proof of Proposition 5

First, under Assumption 5, for $t \geq 2$

$$X_t^* = \sum_{\{h^{t-1} \in H^{t-1}\}} \mu_t^*(h^{t-1}, s) v' \left(\frac{l_t^*(h^{t-1}, u) w_{u,t}^*}{w_{s,t}^*} \right) l_t^*(h^{t-1}, u) \frac{\partial \frac{w_{u,t}^*}{w_{s,t}^*}}{\partial K_{e,t}^*}.$$

Assumption 2 implies $\partial \frac{w_{u,t}^*}{w_{s,t}^*} / \partial K_{e,t}^* < 0$, implying that, in any period $t \geq 2$, $X_t^* < 0$. The first part of the proposition then follows from the first part of Proposition 4 and $X_t^* < 0$, whereas the second part follows from the second part of Proposition 4 and $X_t^* < 0$.

B.5 Proof of Proposition 6

For any t, let $\bar{h}^t = (h^{t-1}, s)$ and for $m \leq t$, let $\bar{h}^m$ be the predecessor in period m. Consider the first-order optimality conditions with respect to $c_t(\bar{h}^t)$ and $l_t(\bar{h}^t)$:

$$
\begin{aligned}
u'(c_t^*(\bar{h}^t))A_t^*(\bar{h}^t) &= \lambda_t^* \pi_t(\bar{h}^t), \\
v'(l_t^*(\bar{h}^t))\left(A_t^*(\bar{h}^t) - \frac{B_t^* \pi_t(\bar{h}^t)}{v'(l_t^*(\bar{h}^t))}\right) &= \lambda_t^* w_t^*(\bar{h}_t)\pi_t(\bar{h}^t),
\end{aligned}
$$

where, letting χ be the indicator function,

$$
\begin{aligned}
A_t^*(\bar{h}^t) &= \pi_t(\bar{h}^t) + \sum_{m=1}^{t} \beta^{t-m}\pi_t(\bar{h}^t|\bar{h}^m)\mu_m^*(\bar{h}^m)\left[\chi_{\{s\}}(\bar{h}_m) - \chi_{\{u\}}(\bar{h}_m)\right] \\
B_t^* &= \sum_{\{h^t \in H^t | h_t = s\}} \mu_t^*(h^t) v'\left(\frac{l_t^*(h^{t-1}, u) w_t^*(u)}{w_t^*(s)}\right) l_t^*(h^{t-1}, u) \frac{\partial \frac{w_t^*(u)}{w_t^*(s)}}{\partial L_{s,t}^*} z_s.
\end{aligned}
$$

By the first order conditions above, $A_t^*(\bar{h}^t) > 0$, $\left(A_t^*(\bar{h}^t) - \frac{B_t^* \pi_t(\bar{h}^t)}{v'(l_t^*(\bar{h}^t))}\right)$ and by Assumption 3, $B_t^* > 0$. These imply

$$
\tau_{y,t}(\bar{h}^t) = 1 - \frac{A_t^*(\bar{h}^t)}{\left(A_t^*(\bar{h}^t) - \frac{B_t^* \pi_t(\bar{h}^t)}{v'(l_t^*(\bar{h}^t))}\right)} < 0. \qquad \square
$$

C Implementation

This section shows how the constrained efficient allocation can be implemented in an incomplete markets environment with taxes. A tax system is said to implement the constrained efficient allocation in a market if the constrained efficient allocation arises as an equilibrium of this market arrangement under the given tax system. The constrained efficient allocation can be implemented in many different ways. We provide an implementation in which the tax system mimics the actual U.S. tax code in the sense that capital tax differentials are created at the firm level through depreciation allowances that differ from actual economic depreciation. Therefore, creating the optimal capital tax differentials would not require further complications to the U.S. tax code.

We begin by describing the market arrangement. Markets are assumed to be incomplete in that agents can only trade non-contingent claims to future consumptions (i.e. they can save and borrow at a net risk-free rate, r_t). An agent's savings are denoted by a_t. There is a representative firm that rents capital at a net interest rate r_t and labor to produce the output good.[24] The wage rates for skilled and unskilled are $w_{s,t}$ and $w_{u,t}$ and are taken as given by the firm.

Government and Taxes. There is a government that needs to finance $\{G_t\}_{t=1}^{\infty}$, an exogenously given sequence of government consumption. The government taxes consumers' savings and labor income (in a nonlinear and history-dependent way). The government also taxes the firms' capital income net of depreciation. The statutory depreciation allowance can differ from economic depreciation, as in the U.S. tax code.

Taxes on consumers are specified following Kocherlakota (2005). Let $\tau_{y,t} : \mathbb{R}_+^t \to \mathbb{R}$ denote the labor tax schedule, where $\tau_{y,t}(y^t)$ is the labor income tax an agent with labor income history $y^t = (y_1, ..., y_t)$ pays in period t. Labor income in history h^t is defined as $y_t(h^t) = w_t(h_t) \cdot l_t(h^t)$. Labor income y_t and labor l_t are one to one for a given level of wages, which means it is sufficient to use either one of them when defining an allocation. In the rest of this section, we use y_t. There are also linear taxes on people's asset holdings, which depend on their income history. Letting $\tau_{a,t} : \mathbb{R}_+^t \to \mathbb{R}$ denote linear tax rates on asset holdings, an agent with income history y^t pays $\tau_{a,t}(y^t)(1 + r_t)a_t(h^{t-1})$, where $a_t(h^{t-1})$ denotes the agent's asset holdings.[25]

Unlike in Kocherlakota (2005), the government also has access to a sequence of linear taxes on firm's capital income, denoted by $(\tau_{f,t})_{t=1}^{\infty}$. The corporate tax code also includes statutory depreciation allowances, $(\bar{\delta}_{s,t}, \bar{\delta}_{e,t})_{t=1}^{\infty}$. The firm is allowed to deduct $\bar{\delta}_{i,t}K_{i,t}$ from

[24] The assumption that the firm does not accumulate capital is innocuous and is made for convenience only. This setup is equivalent to one in which the firm, instead of the consumers, accumulates capital and makes the capital accumulation decisions.

[25] One can show that the government can implement the constrained efficient allocation by taxing only the return from capital $r_t a_t(h^{t-1})$, which would be more in line with what we observe in the U.S. tax code.

its tax base in period t.

Consumer's Problem. Taking prices $(r_t, w_{s,t}, w_{u,t})_{t=1}^{\infty}$ and taxes $(\tau_{y,t}, \tau_{a,t})_{t=1}^{\infty}$ as given, a consumer solves

$$\max_{c,y,a} \sum_{t=1}^{\infty} \sum_{h^t \in H^t} \pi_t(h^t)\beta^{t-1} \left[u(c_t(h^t)) - v\left(\frac{y_t(h^t)}{w_t(h_t)}\right)\right] \qquad \text{s.t}$$

$$c_t(h^t) + a_{t+1}(h^t) \leq y_t(h^t) - \tau_{y,t}(y^t(h^t)) + \left[1 - \tau_{a,t}(y^t(h^t))\right](1 + r_t)a_t(h^{t-1}),$$

$$a_1 = K^*_{s,1} + K^*_{e,1}, \qquad c, y \text{ are nonnegative.}$$

Firm's Problem. Taking prices $(r_t, w_{s,t}, w_{u,t})_{t=1}^{\infty}$ and taxes $(\tau_{f,t}, \bar{\delta}_{s,t}, \bar{\delta}_{e,t})_{t=1}^{\infty}$ as given, the firm solves

$$\max_{K_{e,t}, K_{s,t}, L_{s,t}, L_{u,t}} \widetilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - (1 + r_t)(K_{s,t} + K_{e,t}) - \frac{w_{s,t}}{z_s} L_{s,t} - \frac{w_{u,t}}{z_u} L_{u,t} - \tau_{f,t}\Pi_{f,t},$$

where $\Pi_{f,t}$ is the firm's capital income net of depreciation allowances in period t given by

$$\Pi_{f,t} = \left[F(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - \bar{\delta}_{s,t}K_{s,t} - \bar{\delta}_{e,t}K_{e,t} - \frac{w_{s,t}}{z_s} L_{s,t} - \frac{w_{u,t}}{z_u} L_{u,t} \right].$$

Notice that, as in the U.S. corporate tax code, we assume a flat tax on the firm's capital income net of depreciation allowances.

Equilibrium. Given a tax system $(\tau_{y,t}, \tau_{a,t}, \tau_{f,t}, \bar{\delta}_{s,t}, \bar{\delta}_{e,t})_{t=1}^{\infty}$, an equilibrium is an allocation for consumers, $(c_t(h^t), y_t(h^t), a_{t+1}(h^t))_{t=1}^{\infty}$, an allocation for the firm, $(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t})_{t=1}^{\infty}$, and prices $(r_t, w_{s,t}, w_{u,t})_{t=1}^{\infty}$ such that $(c_t(h^t), y_t(h^t), a_{t+1}(h^t))_{t=1}^{\infty}$ solves the consumer's problem, $(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t})_{t=1}^{\infty}$ solves the firm's problem, and markets clear:

$$\sum_{h^t \in H^t} \pi_t(h^t) c_t(h^t) + K_{s,t+1} + K_{e,t+1} + G_t = \widetilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}),$$

$$K_{s,t} + K_{e,t} = \sum_{h^t \in H^t} \pi_t(h^t) a_t(h^{t-1}),$$

$$L_{s,t} = \sum_{\{h^t \in H^t | h_t = s\}} \pi_t(h^t) \frac{y_t(h^t)}{w_{s,t}} z_s, \qquad L_{u,t} = \sum_{\{h^t \in H^t | h_t = u\}} \pi_t(h^t) \frac{y_t(h^t)}{w_{u,t}} z_u.$$

The government's period-by-period budget balance is implied by Walras' law:

$$\sum_{h^t \in H^t} \pi_t(h^t) \left[\tau_{a,t}(y^t(h^t))(1 + r_t) a_t(h^{t-1}) + \tau_{y,t}(y^t(h^t)) \right] + \tau_{f,t} \Pi_{f,t} = G_t.$$

In what follows, we describe an optimal tax system that implements the constrained efficient allocation in the market setup described above. Before doing so, we provide a formal definition of our notion of implementation.

Implementation. A tax system $(\tau_{y,t}, \tau_{a,t}, \tau_{f,t}, \bar{\delta}_{s,t}, \bar{\delta}_{e,t})_{t=1}^{\infty}$ implements the constrained efficient allocation $(c_t^*(h^t), y_t^*(h^t), K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*)_{t=1}^{\infty}$ if an allocation for consumers $(c_t^*(h^t), y_t^*(h^t), a_{t+1}(h^t))_{t=1}^{\infty}$ and an allocation for the firm $(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*)_{t=1}^{\infty}$ jointly with the tax system and prices $(r_t, w_{s,t}, w_{u,t})_{t=1}^{\infty}$ constitute an equilibrium.

C.1 Optimal Tax System

In this section, we construct the optimal tax system, prove that it implements the constrained efficient allocation, and characterize its properties.

Optimal Tax System. We begin by describing optimal savings taxes. Set the taxes on people's savings as

$$\tau_{a,t+1}^*(y^{t+1}) = 1 - \frac{u'(c_t^*(h^t))}{\beta u'(c_{t+1}^*(h^{t+1})) \tilde{F}_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*)}, \text{ if } y^{t+1} \in Y^{t+1*}$$

$$\tau_{a,t+1}^*(y^{t+1}) = 1, \text{ if else},$$

where $y^t(h^t) = (y_m(h^m))_{m=1}^t, Y^{t*} = \{y^t : y^t = y^{t*}(h^t),\ h^t \in H^t\}$.

In words, Y^{t*} is the set of labor income histories observed at the constrained efficient allocation. Set labor income taxes such that if $y^t \in Y^{t*}$, then $\tau_{y,t}^*(y^t)$ and $a_{t+1}^*(h^t)$ are defined to satisfy the flow budget constraints every period:

$$c_t^*(h^t) + a_{t+1}^*(h^t) = y_t^*(h^t) - \tau_{y,t}^*(y^t) + \left[1 - \tau_{a,t}^*(y^t)\right] \tilde{F}_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) a_t^*(h^{t-1}).$$

If $y^t \notin Y^{t*}$, then $\tau_{y,t}^*(y^t) = 2y_t$.

Finally, set

$$\tau_{f,t}^* = 1 - \frac{F_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) - \delta_s}{F_2(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) - \delta_e}, \qquad \bar{\delta}_{s,t}^* = r_t^* + \delta_s, \qquad \bar{\delta}_{e,t}^* \quad = \quad \delta_e.$$

Observe that at the corporate level, the optimal tax system works in the same way as the U.S. tax code. In the U.S. corporate tax code, there is a single statutory corporate tax rate, and differences between effective tax rates on equipment and structure capital income are created through differences in tax depreciation rules, as explained in detail in Appendix A.

Proposition C.1. *The optimal tax system described above implements the constrained efficient allocation.*

Proof. First, define prices as $r_t^* = F_1(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*) - \delta_s$ and $\forall h \in H, w_{h,t}^* = \frac{\partial F(K_{s,t}^*, K_{e,t}^*, L_{s,t}^*, L_{u,t}^*)}{\partial L_{h,t}} z_h$. Second, observe that the only budget feasible income strategy for the household is the one that corresponds to some agent's income process at the constrained efficient allocation (i.e., for any history h^t in any period t, y^t should be in Y^{t*}).

Third, we claim that if an agent chooses an income strategy y', where this means $y_t'(h^t) = y_t^*(\hat{h}^t)$ for some $\hat{h}^t$, then facing the prices defined above, the agent also chooses (c', a'), meaning that $c_t'(h^t) = c_t^*(\hat{h}^t)$ and $a_{t+1}'(h^t) = a_{t+1}^*(\hat{h}^t)$ for all h^t, t. If this claim can be proved, then the result that agents will actually choose the constrained efficient allocation follows, since the constrained efficient allocation is incentive compatible.

To see this claim, take an agent that follows income strategy y'. His problem is

$$
\begin{aligned}
& \max_{c,a} \sum_{t=1}^{\infty} \sum_{h^t \in H^t} \pi_t(h^t) \beta^{t-1} \left[u(c_t(h^t)) - v\left(\frac{y_t^*(\hat{h}^t)}{w_t^*(h^t)} \right) \right] \qquad \text{s.t} \\
c_t(h^t) + a_{t+1}(h^t) \quad \leq \quad & y_t^*(\hat{h}^t) - \tau_{y,t}^*(y^{t*}(\hat{h}^t)) + \left[1 - \tau_{a,t}^*(y^{t*}(\hat{h}^t)) \right] (1 + r_t^*) a_t(h^{t-1}), \\
& a_1 \leq K_{s,1}^* + K_{e,1}^*, \qquad c \text{ is nonnegative.}
\end{aligned}
$$

The first-order conditions to this problem are the budget constraint with equality and

$$
u'(c_t(h^t)) = (1 + r_{t+1}^*) \beta \sum_{h^{t+1}|h^t} \pi_{t+1}(h^{t+1}|h^t) u'(c_{t+1}(h^{t+1}))[1 - \tau_{t+1}^{a*}(y^{t+1*}(\hat{h}^t, h_{t+1})].
$$

Clearly, the agent's problem is concave, and hence these first-order conditions are necessary and sufficient for optimality provided that a relevant transversality condition holds.

By construction of the labor tax code and the prices, $c_t^*(\hat{h}^t)$ and $a_{t+1}^*(\hat{h}^t)$ satisfy the flow budget constraints. To see that they also satisfy the Euler equation above, observe that wealth taxes are constructed such that

$$
1 - \tau_{a,t+1}^*(y^{t+1*}(\hat{h}^t, h_{t+1})) = \frac{u'(c_t^*(\hat{h}^t))}{\beta u'(c_{t+1}^*(\hat{h}^t, h_{t+1})) \tilde{F}_1^*(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t})}.
$$

Finally, one also needs to make sure that the firm chooses the right allocation. The firm's optimality conditions for labor are satisfied at the constrained efficient allocation by construction of wages. The firm's optimality conditions for capital are

$$
\begin{aligned}
(K_{s,t}) \quad &: \quad \tilde{F}_1(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - (1 + r_t^*) - \tau_{f,t}^* \left[F_1(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - \bar{\delta}_{s,t} \right] = 0 \\
(K_{e,t}) \quad &: \quad \tilde{F}_2(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - (1 + r_t^*) - \tau_{f,t}^* \left[F_2(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}) - \bar{\delta}_{e,t} \right] = 0.
\end{aligned}
$$

At the constrained efficient allocation, the first condition above holds by construction of r_t^* and $\bar{\delta}_{s,t}^*$ whereas the second one holds by construction of $\tau_{f,t}^*$, r_t^* and $\bar{\delta}_{e,t}^*$. □

Properties of Optimal Capital Taxes. Next, we summarize and discuss the properties of optimal capital taxes.

1. Capital income is taxed twice, once at the consumer level via savings taxes and once at the corporate level (double taxation of capital).

2. The statutory corporate tax is strictly positive if only downward incentive constraints bind: $\forall t \geq 2 : \tau^*_{f,t} > 0$, where the inequality comes from Proposition 5.

3. There is a single statutory tax rate on corporate income, τ^f_t, but the effective taxes on capital income at the firm level differ across different capital assets because of differences in the statutory depreciation allowances. The firm is allowed to expense all of its user cost of structure capital, whereas the depreciation allowance on equipment capital is equal to its economic depreciation. These optimal tax depreciation allowances imply that the optimal effective *corporate* tax rate on structure capital is zero and the optimal effective *corporate* tax rate on equipment capital is equal to the statutory corporate tax rate $\tau^*_{f,t}$.[26]

4. Properties 2 and 3 imply that the government uses capital income taxes to collect revenue at the corporate level, as in the U.S. tax code.

5. Expected taxes on consumers' asset holdings are zero as in Kocherlakota (2005):

$$E_t \left\{1 - \tau^{a*}_{t+1}(y^{t+1})\right\} = E_t \left\{ \frac{u'(c^*_t(h^t))}{\beta u'(c^*_{t+1}(h^{t+1}))\tilde{F}^*_{1,t+1}} \right\} = 1.$$

   This property follows from the inverse Euler equation (B.6).

[26]Note that there are other implementations in which both capital types can be taxed positively as long as the tax rates create the efficient capital return wedge. In that case, the savings' tax that the consumers face would have to be adjusted.

D Alternative Tax Systems

D.1 Current Differential Taxation of Capital

In the current DTC, the planner must use the capital income taxes as in the U.S. tax code. This means that the current DTC imposes a set of constraints for both types of agents in each period (here, τ_s and τ_e are fixed over time and taken from the data, namely, $\tau_s = 0.422$ and $\tau_e = 0.371$): for all $h \in H$

$$1 - \tau_s = \frac{\frac{u'(c_{h,t})}{\beta u'(c_{h,t+1})} - 1}{F_{1,t+1} - \delta_s} \text{ and } 1 - \tau_e = \frac{\frac{u'(c_{h,t})}{\beta u'(c_{h,t+1})} - 1}{F_{2,t+1} - \delta_e}, \tag{D.1}$$

Observe that the restrictions that the current capital taxes impose on the set of allocations that the planner can choose, (D.1), take into account that the U.S. tax code taxes capital income net of depreciation. Also, notice that any three of the constraints in (D.1) imply the fourth. Therefore, we write the current DTC planning problem, ignoring the fourth:

$$\begin{aligned}
& \max_{\{(c_{h,t},l_{h,t})_{h\in H},K_{s,t},K_{e,t},L_{u,t},L_{s,t}\}_{t=0}^{\infty}} && \sum_{h\in H} \pi_h \sum_{t=1}^{\infty} \beta^{t-1}\left[u(c_{h,t}) - v(l_{h,t})\right] \qquad \text{s.t.} \\
\forall t \geq 1, \quad & G_t + \sum_h \pi_{h\in H} c_{h,t} + K_{s,t+1} + K_{e,t+1} &\leq\;& \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}), \\
& \sum_{t=1}^{\infty} \beta^{t-1}\left[u(c_{s,t}) - v(l_{s,t})\right] &\geq\;& \sum_{t=1}^{\infty} \beta^{t-1}\left[u(c_{u,t}) - v(\frac{l_{u,t} w_{u,t}}{w_{s,t}})\right], \\
\forall t \geq 1, \quad & \beta u'(c_{s,t+1})\left[(1-\tau_s)(F_{1,t+1} - \delta_s) + 1\right] &=\;& u'(c_{s,t}), \\
\forall t \geq 1, \quad & \beta u'(c_{s,t+1})\left[(1-\tau_e)(F_{2,t+1} - \delta_e) + 1\right] &=\;& u'(c_{s,t}), \\
\forall t \geq 1, \quad & \beta u'(c_{u,t+1})\left[(1-\tau_s)(F_{1,t+1} - \delta_s) + 1\right] &=\;& u'(c_{u,t}).
\end{aligned}$$

D.2 Optimal Nondifferential Taxation of Capital

In optimal NDTC, the planner is not allowed to tax the two types of capital differentially. This means that for agents of both types $h \in H$ and all $t \geq 1$,

$$1 - \tau_{s,t+1}(h) = 1 - \tau_{e,t+1}(h).$$

This restriction on taxes is equivalent to the following restriction on allocations: for all $h \in H$ and $t \geq 1$,

$$1 - \tau_{s,t+1}(h) = \frac{\frac{u'(c_{h,t})}{\beta u'(c_{h,t+1})} - 1}{F_{1,t+1} - \delta_s} = 1 - \tau_{e,t+1}(h) = \frac{\frac{u'(c_{h,t})}{\beta u'(c_{h,t+1})} - 1}{F_{2,t+1} - \delta_e}.$$

This equation can be simplified to an equality constraint on net returns: $F_{1,t+1} - \delta_s = F_{2,t+1} - \delta_e$, which we impose for each period. The optimal NDTC planning problem reads:

$$\begin{aligned}
& \max_{\{(c_{h,t}, l_{h,t})_h, K_{s,t}, K_{e,t}, L_{u,t}, L_{s,t}\}_{t=0}^{\infty}} && \sum_{h=u,s} \pi_h \sum_{t=1}^{\infty} \beta^{t-1} \left[u(c_{h,t}) - v(l_{h,t}) \right] \qquad \text{s.t.} \\
\forall t \geq 1, \quad & G_t + \sum_h \pi_h c_{h,t} + K_{s,t+1} + K_{e,t+1} &\leq\ & \tilde{F}(K_{s,t}, K_{e,t}, L_{s,t}, L_{u,t}), \\
& \sum_{t=1}^{\infty} \beta^{t-1} \left[u(c_{s,t}) - v(l_{s,t}) \right] &\geq\ & \sum_{t=1}^{\infty} \beta^{t-1} \left[u(c_{u,t}) - v(\frac{l_{u,t} w_{u,t}}{w_{s,t}}) \right], \\
\forall t \geq 1, \quad & F_1(K_{s,t+1}, K_{e,t+1}, L_{s,t+1}, L_{u,t+1}) - \delta_s &=\ & F_2(K_{s,t+1}, K_{e,t+1}, L_{s,t+1}, L_{u,t+1}) - \delta_e.
\end{aligned}$$

References for the Appendix

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